जननायक चन्द्रशेखर विश्वविद्यालय, बलिया

JANANAYAK CHANDRASHEKHAR UNIVERSITY, BALLIA

(A State University established under the Uttar Pradesh University Act 1973)

M. Sc. MATHEMATICS

Syllabus(w.e.f. 2024-25)

Programme Name: M.Sc. Mathematics

Programme Code: PG MAT 100

With the growing role of Science and Technology in our lives, the importance of Mathematics is also increasing. A strong knowledge in mathematics has become essential for careers in the field of STEM (Science, Technology, Engineering and Mathematics). Moreover, use of Mathematics is increasing in Management, Industry, Finance, Banking and other fields.

M.Sc. Mathematics programme at Jananayak Chandrashekhar University aims at giving students a strong foundational knowledge in advanced mathematics. The programme is structured in such a manner that students can then proceed to pursue research in Mathematics as well as be skilled enough to pursue other careers requiring mathematical knowledge.

Programme Outcomes: After successfully completing this programme the student will be able to

1. Explain various concepts related to advanced mathematics.
2. Apply the knowledge gained to solve real world problems in the fields of Science, Technology, Management, Industry etc.
3. Formulate new theories and do research in mathematics.

Programme Structure:

1. The Post-Graduation programme in Mathematics of this University comprises of four semesters.
2. Each semester has four theory courses (papers), each of 5 credits and 100 marks.
3. Along with this, each student has to do one research project of 4 credits in each semester.
4. The reports of the projects carried out in 1st and 2nd semester will be compiled together and submitted in the form of Project Report/ Dissertation at the end of second semester. It will be evaluated out of 100 marks.
5. Similarly the reports of the projects carried out in 3rd and 4th semester will be compiled together and submitted in the form of Project Report/ Dissertation at the end of fourth semester. It will be evaluated out of 100 marks.
6. In 1st or 2nd semester, the student will have to study one minor elective course from a different faculty which will be of 4/5 credit.

The outline of the M.A./M.Sc. Programme is given on the following page.

M.A./M.Sc. MATHEMATICS

SEMESTER-1					
Paper	**Paper/Course Code**	**Paper/Course Name**	**Marks**	**Credits**	**Hours**
I	MAT 101	Algebra – I	100	5	75
II	MAT 102	Real Analysis	100	5	75
III	MAT 103	Basic Topology	100	5	75
IV	MAT 104	Complex Analysis	100	5	75
V	MAT 105	Project		4	

Note: Students will have to select one minor elective paper from other faculty (faculty other than Science) also.

SEMESTER-2					
I	MAT 201	Algebra – II	100	5	75
II	MAT 202	Functional Analysis – I	100	5	75
III	MAT 203	Measure & integration – I	100	5	75
IV	**Elective (Optional) (Any one of the following)**				
	MAT 204A	Classical Mechanics	100	5	75
	MAT 204B	Special theory of Relativity	100	5	75
V	MAT 205	Project	100 (Sem-1 +Sem-2)	4	

SEMESTER-3					
I	MAT 301	Topology	100	5	75
II	MAT 302	Differential & Integral Equations	100	5	75
III **Elective (Optional) Papers (Any one of the following)**					
	MAT 303A	Differential Geometry of Manifolds	100	5	75
	MAT 303B	Hydrodynamics	100	5	75
IV- **Elective (Optional) Papers (Any one of the following)**					
	MAT 304A	Operations Research	100	5	75
	MAT 304B	Advanced Linear Algebra	100	5	75
V	MAT 305	Project		4	

SEMESTER-4					
I	MAT 401	Functional Analysis - II	100	5	75
II	MAT 402	Measure & Integration - II	100	5	75
III- **Elective (Optional) Papers (Any one of the following)**					
	MAT 403A	Complex manifolds & Contact manifolds	100	5	75
	MAT 403B	Fluid Mechanics	100	5	75
IV- **Elective (Optional) Papers (Any one of the following)**					
	MAT 404A	General Relativity and Cosmology	100	5	75
	MAT 404B	Theory of optimization	100	5	75
	MAT 404C	Mathematical modeling	100	5	75
V	MAT 405	Project	100 (Sem-3 + Sem-4)	4	

SEMESTER – I

MAT 101 **ALGEBRA – I** **Credit-5/ Hours-75**

Course Objective: students will learn concepts related to advanced algebra and how to apply them in solving mathematical problems.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Explain various concepts related to algebra.
2. Apply results in problems arising in higher mathematics.
3. Characterize different types of groups.

UNIT I:

Action of a group G on a set S, Equivalent formulation as a homomorphism of G to T(S), Examples, Stabilizer (Isotropy) subgroups and Orbit decomposition, Class equation of an action, Its particular cases (left multiplication and conjugation), Conjugacy class equation, Core of a subgroup.Sylow's Theorem I, II and III,

UNIT II:

Subnormal and normal series, Zassenhaus's lemma (Statement only)Schreier's refinement theorem, composition series, Jordan – Holder theorem, Chain conditions, Examples, Internal and External direct products and their relationship, Indecomposability. p–groups, Examples and applications, Groups of order pq.

UNIT III:

Commutators, Solvable groups, Solvability of subgroups, factor groups and of finite p – groups, Examples, Lower and upper central series, Nilpotent groups and their equivalent characterizations.

UNIT IV:

Factorization theory in commutative domains, Prime and irreducible elements, G.C.D., Euclidean domains, Maximal and prime ideals, Principal ideal domains, Unique factorization domains, Examples and counter examples, Chinese remainder theorem for rings and PID's, Polynomial rings over domains, Eisenstein's irreducibility criterion, Unique factorization in polynomial rings over UFD's.

Reference Books:

1. Dummit,D.S. and Foote, R. M.(2003). *Abstract Algebra*.John Wiley, N.Y.
2. Gopalakrishnan, N.S.(2015).*University Algebra* (3rd ed.).New Age Int. Pub.
3. Jacobson, N.(1984). *Basic Algebra*(Vol. 1). Hindustan Publishing Co, New Delhi.
4. Lal, R.(2002). *Algebra*(Vols. I & II). Shail Publications, Allahabad.

MAT 102 **REAL ANALYSIS** **Credit-5/ Hours-75**

Course Objective: students will learn about concepts related to real analysis such as R-S integration, convergence of sequence and series of functions and multivariable calculus and their applications in mathematics and science.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Explain concepts like R-S integration, convergence of sequence of functions, multivariable calculus etc.
2. Solve problems real analysis in Mathematics and Sciences.
3. Compare different types of integrations.
4. Do research using knowledge gained in this course.

UNIT I:

Definition and existence of Riemann – Stieltjes integral, Conditions for R–S integrability. Properties of the R-S integral, R-S integrability of functions of a function Integration and differentiation, Fundamental theorem of Calculus.

Unit II:

Series of arbitrary terms. Convergence, divergence and oscillation, Absolute Convergence, Abel's and Dirichilet's tests. Multiplication of series. Rearrangements of terms of a series, Riemann's theorem and sum of series, Sequences and series of functions.

Unit III:

Pointwise and uniform convergence, Cauchy's criterion for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjies integration, Uniform convergence and differentiation, Weierstrass approximation theorem, Power series. Uniqueness theorem for power series, Abel's and Tauber's theorems.

Unit IV:

Functions of Several Variables, Linear transformations, Derivatives in an open subset of R^n Jacobian matrix and Jacobians, Chain rule and its matrix form, Interchange of order of differentiation, Derivatives of higher orders Taylor's theorem, Inverse function theorem, Implicit function theorem, Extremum problems with constraints, Lagranges multiplier method.

Reference Books:

1- Rudin, W.(1976). *Principles of Mathematicsl Analyis* (2nd ed.). McGraw-Hill International Student Edition.

2- Apostol, T. M.(1985). *MathematicalAnalysis*. Narosa Publishing House, New Delhi.

3- Lang, S. (1969). *Analysis I and II*. Addision-Wesley Pub. Co.

MAT 103 **BASIC TOPOLOGY** **Credit-5/ Hours-75**

Course Objective: students will learn about concepts related to metric spaces and topological spaces and how different types of topological spaces have different properties.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Explain various concepts related to Metric Space and Topology.
2. Characterize different types of Topological spaces.
3. Solve problems requiring knowledge of Topology.

UNIT I:

Metric spaces: Continuity of functions, Properties of continuous functions, Homeomorphisms. Connectedness in metric spaces, Connected sets in the real line, Continuity and connectedness, Compactness, closed subset of a compact space, compact subset of a metric space, Continuity and compactness.

UNIT II:

Definition and examples of topological spaces. Closed sets. Closure. Dense sets. Neighborhoods, interior, exterior, and boundary. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology.

UNIT III:

Alternative methods of defining a topology in terms of Kuratowski closure operator and neighborhood systems. Continuous functions and homeomorphism. First and second countable spaces. Lindelof spaces. Separable spaces. Nets and filters.

UNIT IV:

The separation axioms $T_0, T_1, T_2, T_3, T_{3\frac{1}{2}}, T_4$; their characterizations and basic properties. Urysohn's lemma, Tietze extension theorem. Metric topology and metrization.

Reference Books:

1. Kelley, J. L.(1995). *General Topology*. Van Nostrand.
2.Joshi, K. D.(1983). *Introduction to General Topology*. Wiley Eastern.
3.Munkres, J. R.(2000).*Topology*(2nd ed.). Pearson Internationl.
4.Dugundji, J. (1966). *Topology*. Prentice-Hall of India.

MAT 104 **COMPLEX ANALYSIS** **Credit-5/ Hours-75**

Course Objective: students will learn various concepts related to complex analysis and their application in mathematics and science.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Explain various concepts related to complex analysis.
2. Apply results studied in problems arising in physical sciences.
3. Relate different theorems.
4. Develop new theories and do research in higher mathematics.

UNIT I:

Analytic continuation, uniqueness of analytic continuation, Natural Boundary, complete analytic functions , Power series method ofAnalytic continuation, Schwarz's Lemma, Inverse function theorem, Schwarz's reflection principle, Reflection across analytic arcs.

UNIT II:

Residue at infinity, Cauchy's Residue theorem, Contour integration: Integral of the type$\int_{\alpha}^{2\pi+\alpha} f(cos\theta, sin\theta)d\theta$,$\int_{-\infty}^{\infty} f(x)dx$,$\int_{-\infty}^{\infty} g(x)cosmx\, dx$. Singularities on the real axis, Integrals involving branch points, Jordan's Lemma.

UNIT III:

The Riemann mapping theorem, Behavior at the boundary, Picard' theorem, Borel theorem, Infinite Products, Jensen's formula, Poission –Jenson formula, Borel Cartheodory theorem.

UNIT IV:

Entire Functions with Rational Values, The Phragmen-Lindelof and Hadamard Theorems, Meromorphic Functions, Mittag-Leffler Theorem,Weierstrass factorization theorem, Gama functions.

Assignments:

1. Discuss power series method of analytic continuation.
2. State and prove Cauchy's Residue theorem.
3. Prove the Riemann mapping theorem.
4. Write a note on Merormophic functions.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Lang, S.(1999).*Complex Analysis* (4th ed.). Springer.

2.Bak, J. and Newman, D. J.(2010).*Complex Analysis* (3rd ed.). Springer.

3. Conway, J. B.(1980).*Functions of One Complex Variable*(2nd ed.).Narosa Pub. House.

MAT 105 **PROJECT** **Credit-4**

Each student will have to complete a project in first semester. It will be of 4 credits. The evaluation of Semester I and Semester II projects will be done together at the end of second semester.

SEMESTER – II

MAT 201 **ALGEBRA – II** **Credit-5/ Hours-75**

Course Objective: students will learn about algebraic concepts related to module theory and field theory and their applications.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Describe properties of Modules and Fields.
2. Differentiate different types of Modules.
3. Use these results in advanced level mathematical studies.

UNIT I:

Modules over a ring. Endomorphism ring of an abelian group. R-Module structure on an abelian group M as a ring homomorphism form R to End M, Submodules, Direct summands, Homomorphism, Factor modules, Correspondence theorem, Isomorphism theorems, Exact sequences, Five lemma.

UNIT II:

Free modules, Homomorphism extension property, Equivalent characterization as a direct sum of copies of the underlying ring, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules.

UNIT III:

Noetherian modules and rings, Equivalent characterizations, Submodules and factors of noetherian modules, Characteristic of a field, Prime subfields, Field extension, Finite extensions, Algebraic and transcendental extensions. Factorization of polynomials in extension fields, Splitting fields and their uniqueness.

UNIT IV:

Separable field extensions, Perfect fields, Separability over fields of prime characteristic, Transitivity of separability, Automorphisms of fields, Dedekind's theorem, Fixed fields, Normal extensions, Splitting fields and normality, Normal closures.

Reference Books:

1. Dummit, D. S. and Foote, R. M.(2003). *Abstract Algebra*. John Wiley, N.Y.

2. Anderson F. W. and Fuller, K. R.(1974). *Rings and Categories of Modules*, Springer, N.Y.

3. Adamson, I. A.(1964). *An Introduction to Field Theory*. Oliver & Boyd, Edinburgh.

4. Gopalakrishnan, N. S. (2015). *University Algebra* (3rd ed.).New Age Int. Pub.

5. Hungerford, T. W.(2004). *Algebra*. Springer (India) Pvt. Ltd.

6. Lal, R. (2002). *Algebra* (Vol. 2.). Shail Publishing House, Allahabad.

MAT: 202 **FUNCTIONAL ANALYSIS - I** **Credit-5/ Hours-75**

Course Objective: students will learn about normed linear spaces, their properties and types and functions on them.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Give examples of Normed linear spaces.
2. Differentiate between different types of normed linear spaces.
3. Use the results in problems arising in Functional Analysis and other branches of mathematics.

UNIT I:

Normed linear spaces, Banach spaces, their examples including $\mathbb{R}^n$, $\mathbb{C}^n$,l_p^n , l_p, C[a,b] and topological properties, Holder's and Minkowski's inqualities, Subspaces,.Quotient space of a normed linear space and its completeness.

UNIT II:

Continuous linear transformations, Spaces of bounded transformations, Continuous linear functional, Hahn Banach theorems(separation and extension), strict convexity and uniqueness of Hahn Banach extension , Banach Steinhous theorem, Uniform boundedness principle.

UNIT III:

Open mapping theorem, Bounded inverse theorem, Projection,Closed graph theorem, Finite dimensional normed linear spaces, Compactness, Equivalent norms, Bolzano Weistrass property.

UNIT IV:

Duals of $\mathbb{R}^n$, $\mathbb{C}^n$,l_p^n, l_p, C[a,b], weak and weak* convergence, Embedding and reflexivity, Uniform convexity and Milman theorem.

Reference Books:

1. Simmons, G. F. (1963). *Introduction to Topology and Modern Analysis*. McGraw Hill.
2. Ponnusamy, S. (2002). *Foundation of Functional Analysis*. Narosa Publishing House.
3. Limaye, B. V.(2017).*Functional Analysis*(3rd ed.). New Age Int. Publisher.

MAT 203 **MEASURE AND INTEGRATION – I** **Credit-5/ Hours-75**

Course Objective: students will learn concepts related to Lebesgue measure and Lebesgue integration and their applications.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Compare different types of integration.
2. Select the appropriate results to use them in solving problems.
3. Use the results in Mathematical research.

UNIT I:

Lebesgue outer and inner measure, Lebesgue measure on R, translation invariance of Lebesgue measure, existence of a non-measuable set, characterizations of Lebesgue measurable sets, Borel sets, Cantor-Lebesgure function.

UNIT II:

Measurable functions on a measure space and their properties, Borel measurable functions, simple functions and their integrals, Lebesgue integral on R and its properties, Riemann and Lebesgue integrals.

UNIT III:

Integral of non negative measureable function and of unbounded functions,Bounded convergence theorem, Fatou's lemma, Monotone convergence theorem, Lebesgue dominated convergence theorem.

UNITIV:

The L^p-space. Convex functions. Jensen's inequality, Holder and Minkowski inequalities, Completeness of L^p, Convergence in measure, Almost uniform convergence.

Reference Books:

1. Royden, H.L. and Fitzpatrick, P.M.(2015).*Real Analysis*(4th ed.). Pearson.
2. Halmos, P.R.(1994). *Measure Theory*. Springer.
3. Rana, I.K.(2005). *An Introduction to Measure and Integration* (2nd ed.).Narosa Publishing House.
4. Hewit, E. and Stromberg,K.(1975). *Real and Abstract Analysis*. Springer.

Elective Papers (Optional Papers) (Any one from MAT 204A and MAT 204B)

MAT 204A **CLASSICAL MECHANICS** **Credit-5/ Hours-75**

Course Objective: students will learn concepts related to Classical Mechanics and their application in solving problems related to physical sciences.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Describe concepts related to mechanics.
2. Apply the results in solving problems in physical sciences.
3. Create new theories related to mechanics.

UNIT I:

The linear momentum and the angular momentum of a rigid body in terms of inertia constants, kinetic energy of a rigid body, equations of motion, examples on the motion of a sphere on horizontal and on inclined planes. Euler's equations of motion, motion under no forces, Eulerian angles and the geometrical equations of Euler.

UNIT II:

Generalized co-ordinates, holonomic and non-holonomic systems, configuration space, Lagrange's equations using D'Alembert's Principle for a holonomic conservative system, deduction of equation of energy when the geometrical equations do not contain time explicitly, Lagrange's multipliers case, deduction of Euler's dynamical equations from Lagrange's equations.

UNIT III:

Theory of small oscillations, Lagrange's method, normal (principal) co-ordinates and the normal modes of oscillation, small oscillations under holonomic constraints,stationary property of normal modes, Lagrange equations for impulsive motion.

UNIT IV:

Generalized momentum and the Hamiltonian for a dynamical system, Hamilton's canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, phase space and Hamilton's Variational principle, the principle of least action, canonical transformations, Hamilton-Jacobi theory, Integrals of Hamilton's equations and Poisson- Brackets, Poisson- Jacobi identity.

Reference Books:

1. Ramsey, A. S. (1985). *Dynamics: Part II.* CBS Publishers & Distributors.
2. Goldstein, H. (1969). *Classical Mechanics*. Addison-Wesley Publishing Company.
3. Rana, K. C. and Joag, P. C.(1991) *Classical Mechanics*. Tata McGraw- Hill.

MAT 204B **SPECIAL THEORY OF RELATIVITY** **Credit-5/ Hours-75**

Course Objective: students will learn concepts related to special theory of relativity and their applications.

Learning Outcomes: After successful completion of the syllabus, learners will be able to:

1. Explain concepts related to Special Theory of Relativity.
2. Compare Newtonian Mechanics and Relativistic Mechanics.
3. Apply the results to solve problems related to relativity.

UNIT I:

Review of Newtonian Mechanics, Inertial frame, Speed of light and Galilean relativity, Michelson-Morley experiment, Lorentz-Fitzerold contraction hypothesis, relative character of space and time, postulates of special theory of relativity, Lorentz transformation equations and geometrical interpretation, Group properties of Lorentz transformations.

UNIT II:

Relativistic kinematics, composition of parallel velocities, length contraction, time dilation, transformation equations, equations for components of velocity and acceleration of a particle and contraction factor.

UNIT III:

Geometrical representation of space time, four dimensional Minkowskian space of special relativity, time-like intervals, light-like and space-like intervals, Null cone, proper time, world line of a particle, four vectors and tensors in Minkowskian space time.

UNIT IV:

Relativistic mechanics-Variations of mass with velocity, equivalence of mass energy, transformation equation for mass, momentum and energy, Energy momentum for light vector, relativistic force and transformation equation for its components, relativistic Lagrangian and Hamiltonian, relativistic equations of motion of a particle, energy momentum tensor of a continuous material distribution.

Reference Books:

1. Mollar, C.(1952). *Theory of relativity*, Clarendon press.
2.Resnick; R. (1972).*Introduction to special relativity*. Wiley Eastern Pvt. Ltd.
3.Anderson, J. L.(1967). *Principles of relativity*, Academic Press.

MAT 205 **PROJECT** **Credit-4**

Each student will have to complete a project in Second semester. It will be of 4 credits. The evaluation of Semester I and Semester II projects will be done together at the end of second semester and will consist of 100 marks.

SEMESTER – III

MAT 301 **TOPOLOGY** **Credit-5/ Hours-75**

Course Objective: Students will learn advanced topics related to topology and how to apply those concepts.

Learning Outcomes: After studying this course, students will be able to-

1. Describe various concepts related to Topological Spaces.
2. Use the results in solving problems related to Topology.
3. Correlate different results.

Unit I:

Compactness. Basic properties of compactness. Compactness and finite intersection property. Sequential, countable, and B-W compactness. Local compactness. One-point compactification. Connected spaces and their basis properties. Components. Locally connected spaces. Continuity and connectedness.

Unit II:

Tychonoff product topology in terms of standard sub-base and its characterizations. Product topology and separation axioms, connected-ness, andcompactness (incl. the Tychonoff's theorem), product spaces.

Unit III:

Homotopy of paths, the fundamental group, covering spaces, fundamental group of circle, punctured plane, n-sphere, figure 8 and of surfaces.

Unit IV:

Essential and Inessential maps, equivalent conditions, Fundamental theorem of algebra, Vector fields and fixed points, Brouwer fixed point theorem for disc, Homotopy type and Jordan separation Theorem..

Reference Books:

1.Kelley, J. K. (1995). *General Topology*. Van Nostrand.
2. Joshi, K. D. (1983). *Introduction to General Topology*. Wiley Eastern.
3.Munkres, J. R.(2000). *Topology* (2nd ed.). Pearson International.
4.Dugundji, J. (1966). *Topology*. Prentice-Hall of India.

MAT 302 DIFFERENTIAL AND INTEGRAL EQUATIONS Credit-5/ Hours-75

Course Objective: Students will learn concepts related to Differential equations, integral equations, including methods of and their solutions and their applications in solving mathematical problems.

Learning Outcomes: After studying this course, students will be able to-

1. Differentiate between types of differential equations
2. Use appropriate method to solve differential equations and integral equations.
3. Create solutions of real world problems.

UNIT I:

Linear independence and Wronskians, Initial and Boundary Value Problems, Picards iterations, Lipschitz conditions, Sufficient conditions for being Lipschitzian, Examples of Lipschitzian and non-Lipschitzian functions, Picard's Theorem for local existence and uniqueness of solutions of an initial value problem of first order which is solved for the derivative, examples of problems without solutions and of equations where Picard's iterations do not converge.

UNIT II:

Pfaffian differential equations: Necessary and sufficient conditions for integrability of Total differential equation, Methods for finding solutions-by inspection, Solution of homogeneous equation, Use of auxiliary equations, solution by taking one variable as constant. Non integrable equations.
Orthogonal and Orthonormal sets of functions, Gram Schmidth orthonormalization process, Generalized Fourier Series, Sturm-Liouville problems, Examples of Boundary value problems which are not Sturm-Liouville problems.

UNIT III:

Method of separation of variables: Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, Boundary value problems for transverse vibrations in a string of finite length and heat diffusion in a finite rod, Classification of linear integral equations, Relation between differential and integral equations.

UNIT IV:

Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Reference Books:

1. Sneddon, I. N.(1957).*Elements of Partial Differential Equations*. McGraw-Hill.
2. Amaranath, T.(2003).*An Elementary Course in Partial Differential Equations*. Narosa Pub.
3. Kanwal, R. P.(1997).*Linear Integral Equations*. Birkhäuser,Inc..
4. Raisinghania, M. D. (2018). *Advanced Differential Equations* (19th ed.). S. Chand.
5. Kreyszig, E. (2001). *Advanced Engineering Mathematics* (8th ed.). Wiley India.

Elective Papers(Optional Papers) (Any one from MAT 303A, MAT 303B)

MAT 303A DIFFERENTIAL GEOMETRY OF MANIFOLDS Credit-5/ Hours-75

Course objective: Students will learn concepts related to differential manifolds and the application of these concepts in solving problems related to manifolds.

Learning Outcomes: After studying this course, students will be able to-

1. Describe various concepts related to Manifolds.
2. Compare different types of Manifolds.
3. Use the results to solve problems related to manifolds.

UNIT I:

Definition and examples of differentiable manifolds. Tangent spaces. Vector fields, Jacobian map Lie derivatives. Exterior algebra. Exteriorderivative, Liegroups and Lie algebras.

UNIT II:

Riemannian manifolds, Riemannian connections, Curvature tensors, Sectional curvature, Shur's theorem, Projective curvature tensor , Conformal curvature tensor, Conharmonic curvature tensor and Concircular curvature tensor.

UNIT III:

Homomorphism and isomorphism. Lie transformation groups,Principle fibre bundle, Linear fame bundle, Associated fibre bundle, Vector bundle, Tangent bundle, Induced bundle, Bundle homomorphism.

UNIT IV:

Submanifolds and Hypersurfaces, Normals, Induced connection, Gauss formulas, Weingarten formulae, Lines of curvature, Mean curvature, Generalized Gauss and Minardi-Codazzi's equations.

Reference Books:

1. Mishra, R. S.(1965). *A course in tensors with applications to Riemannian Geometry*.Pothishala (Pvt.) Ltd.

2.Mishra, R. S.(1984).*Structures on a differentiable manifold and their applications*. Chandrama Prakashan, Allahabad.

3. Sinha, B. B. (1982).*An introduction to modern differentrial geometry*. Kalyani Publishes, New Delhi.

MAT 303B **HYDRODYNAMICS** **Credit-5/ Hours-75**

Course objective: Students will learn concepts related to motion of bodies in liquids and application of these concepts in modeling and solving motion of bodies in liquids.

Learning Outcomes: After studying this course, students will be able to-

1. Explain concepts related to motion of bodies in fluids.
2. Apply this knowledge in real world problems.
3. Compare motion of different type of bodies in fluids.

UNIT I:

Equation of continuity, Boundary surfaces, streamlines, Velocity potential, Irrotational and rotational motions, Vortex lines, Euler's Equation of motion, Bernoulli's theorem, Impulsive actions.

UNIT II:

Motion in two-dimensions, Conjugate functions, Source, sink, doublets and their images, Conformal mapping, Circle Theorem.

UNIT III:

Two- dimensional irrotational motion produced by the motion of circular cylinder in an infinite mass of liquid, theorem of Blasius, Motion of Eliptic Cylinder.

UNIT IV:

Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Equation of motion of a sphere. Concentric Spheres.

Reference Books:

1.Besant W. H. and Ramsey, A. S.(1988). *A Treatise on Hydrodynamics*. CBS Pub. Delhi.
2.Yuan, S. W. (1988). *Foundations of Fluid Dynamics*. Prentice-Hall of India.
3.Chorlton,F .(2009) .*Fluid Dynamics*. G. K.Publishers.

Elective Papers(Optional Papers) (Any one from MAT 304A, MAT 304B)

MAT 304A **OPERATONS RESEARCH** **Credit- 5/ Hours-75**

Course objective: Students will learn concepts related to Operations Research like Networks, sequencing, inventory control and nonlinear optimization and their usage in solving problems related to management.

Learning Outcomes: After studying this course, students will be able to-

1. Compare different types of optimization models.
2. Apply optimization techniques to solve problems arising in management.
3. Develop new models of real world problems.

UNIT I:

Network Analysis- Minimal spanning tree, shortest path problem, Maximal Flow problem. Critical path method, Network representation, Forward pass and backward pass, slack and float, Time cost trade off. PERT, Requirements for application of PERT technique, Practical limitations in using PERT, Differences between PERT and CPM.

UNIT II:

Sequencing Problems- n jobs through 2 machines, n jobs through 3 machines, n jobs through m machines, 2 jobs through m machines. Replacement problems (Individual and group).

UNIT III:

Inventory Control: Introduction, Classification of Inventory, Economic parameter associated with inventory problems, Deterministic and Probabilistic models with and without lead time.
Queuing Theory: Structure of queuing system, Single Server Models(M/M/1), Multiple server models(M/M/C), Self-service model.

UNIT IV:

Non-Linear Programming: Introduction and definitions. Formulation of non-Linear programming problems, General non-linear programming problems. Kuhn-Tucker conditions, Lagrangian Method, Constrained optimization with equality constraints. Constrained optimization with inequality constraints. Saddle point problems Saddle points and NLPP. Wolfe's and Beale's method to solve Quadratic Programming problem.

Reference Books:

1. Taha, H. A.(2011). *Operations Research- An Introduction* (9th ed.).Pearson.
2. Rao, S. S.(1978) *Optimization- Theory and Applications*. Wiley Eastern Ltd.
3. Sharma, J.K.(2016) *Operations Research-Theory and Applications*(6th ed.). Trinity Press.

MAT 304B **ADVANCED LINEAR ALGEBRA** **Credit-5/ Hours-75**

Course objective: Students will learn advanced level concepts related to Linear algebra.

Learning Outcomes: After studying this course, students will be able to-

1. Explain different results related to advanced Linear Algebra.
2. Use the results in solving problems related to module theory and matrices.
3. Develop new theories and do research in Linear Algebra.

UNIT I:

Algebraic and geometric multiplicities of eigenvalues, Invariant subspaces, T-conductors and T-annihilators, Minimal polynomials of linear operators and matrices, Characterization of diagonalizability in terms of multiplicities and also in terms of the minimal polynomial, Triangulability, Simultaneous triangulation and diagonalization.

UNIT II:

Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into T(M) and a free module, p-primary components, Decomposition of p-primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

UNIT III:

Reduction of matrices over polynomial rings over a field,Similarity of matrices and F[x]-module structure, Projections, Invariant direct sums, Characterization of diagonalizability in terms of projections, Primary decomposition theorem.

UNIT IV:

Diagonalizable and nilpotent parts of a linear operator, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Semisimple operators, Taylor formula.

Reference Books:

1. Hofmann, K. and Kunze, R.(1971). *Linear Algebra* (2nd ed.).Pearson.
2. Dummit D. S. and Foote, R. M. (2003). *Abstract Algebra*. John Wiley & Sons.
3. Helson, H.(1994). *Linear Algebra*. Hindustan Book Agency.
4. Jacobson, N.(1984). *Basic Algebra*(Vol. 1). Hindustan Publishing Co.
5. Gopalakrishnan, N. S. (2015). *University Algebra* (3rd ed.). New Age Int. Pub.
6. Hungerford, T. W.(2004). *Algebra*. Springer (India) Pvt. Ltd.
7. Musili, C.(1994). *Rings and Modules*. Narosa Publishing House.

MAT 305 **PROJECT** **Credit-4**

Each student will have to complete a project in Third semester. It will be of 4 credits. The evaluation of Semester- III and Semester-IV projects will be done together at the end of fourth semester and will consist of 100 marks.

SEMESTER- IV

MAT 401 **FUNCTIONAL ANALYSIS - II** **Credit- 5/ Hours-75**

Course objective: Students will learn concepts related to Hilbert spaces and their application in solving mathematical problems.

Learning Outcomes: After studying this course, students will be able to-

1. Give examples to Inner Product Spaces.
2. Explain concepts related to Hilbert spaces.
3. Use the results in solving problems arising in Functional Analysis.

UNIT I:

Inner product spaces with example, Polarization identity, Scwartz inequality, Parallelogram law, Uniform convexity of norm induced by inner product,Orthonarmal sets, Gram-Schmidt Orthogonalisation, Hilbert spaces.

UNIT II:

Bessel's inequality, Riesz-Fisher theorem, orthonormal basis, characterization of orthonormal basis, Fourier series representation and Parsevell's relation, Separable Hilbert paces, Continuity of linear mappings, Projection theorem, Riesz- representation theorem, reflexivity of a Hilbert's space, Unique Hahn extension theorem, weak convergence and weak boundedness.

UNIT III:

Unitary operators on a Hilbert spaces, Adjoint of an operator, Self adjoint and normal operators with examples, Characterization and results pertaining to these operators, Positive operator, Shift operator, Projection on a Hilbert's space.

UNIT IV:

Finite dimensional spectral theory, Determinant and spectrum of an operator, Spectral theorem, spectral resolution.

Reference Books:

1. Simmons,G. F.(1963).*Introduction to Topology and Modern Analysis*. McGraw Hill(India).
2. Ponnusamy, S.(2002).*Foundations of Functional Analysis*. Narosa Publishing House.
3. Limaye, B. V.(2017). *Functional Analysis*(3rd ed.). New Age Int. Publication.

MAT 402: **MEASURE AND INTEGRATION - II** **Credit-5/ Hours-75**

Course objective: Students will learn concepts related to σ- algebras, signed measures, product measure, their properties and use in integration.

Learning Outcomes: After studying this course, students will be able to-

1. Describe different types of measures and their properties.
2. Solve problems related to measure and integration.
3. Relate various results.

UNIT I:

Semialgebras, algebras, monotone class, σ- algebras, measure and outer measures, Caratheodory extension process of extending a measure on a semi-algebra to generated σ –algebra, completion of measure space.

UNIT II:

Signed measure. Hahn and Jordan decomposition theorems. Absolutely continuous and singular measures. Radon Nikodyn theorem. Lebesgue decomposition. Riesz-Representation theorem, Extension theorem (carathedory).

UNIT III:

Product measures, Fubini's theorem, Baire sets, Baire measure, Continuous functions with compact support.

UNIT IV:

Regularity of measures on locally compact spaces. Integration of continuous functions with compact support. Riesz-Markov theorem.

zReference Books:

1.Royden, H. L. and Fitzpatrick, P.M.(2015).*Real Analysis*(4th ed.). Pearson.
2.Halmos, P. R.(1950). *Measure Theory*. Van Nostrand.
3.Berberian, S. K.(1981). *Measure and Integration*. Wiley Eastern.
4.Taylor,A. E.,(1958). *Introduction to Functional Analysis*. John Wiley.
5. Barra, G. D.(1981). *Measure Theory and Integration*. Wiley Eastern.
6. Bartle, R. G. (1966). *The Elements of Integration*. John Wiley.
7. Rana, I. K. (2005). *An Introduction to measure and Integration* (2nd ed.). Narosa Publishing House.

Elective Papers(Optional Papers) (Any one from MAT 403A, MAT 403B)

MAT 403A COMPLEX MANIFOLDS AND CONTACT MANIFOLDS **Credit-5/ Hours-75**

Course objective: Students will learn concepts related to complex manifolds, contact manifolds Sasakian manifolds etc.

Course Outcomes: After studying this course, students will be able to-

1. Explain concepts related to complex Manifolds and contact Manifolds.
2. Differentiate between different Manifolds.
3. Apply the results to solve problems.
4. Develop new theories and do research in Manifolds.

UNIT I:

Almost complex manifolds: Elementary notions ,Nijenuis tensor, Eigen values of F, Integrability conditions, Contravariant and covariant almost analytic vectors fields, F connection.

UNIT II:

Almost Hermite manifolds: Definition, Curvature tensor, Linear connection, Kaehler manifolds: Definition, Curvature tensor, Properties of Projective, Conformal, Conharmonic and concircular curvature tensor.

UNIT III:

Almost contact manifolds: Definition, Eigen values of F, Lie derivative, Normal contact structure, Particular affine connection, Almost Sasakian manifold.

UNIT IV:

Sasakian manifolds: K- contact Riemannian manifold and its properties , Sasakian manifolds and its properties, Properties of Projective, Conformal, Conharmonic and concircularcurvature tensor in Sasakian manifolds; Cosymplectic structure.

Reference Books:

1.Mishra, R.S.(1965).*A course in tensors with applications to Riemannian Geometry*. Pothishala (Pvt.)Ltd.
2.Mishra, R. S.(1984).*Structures on a differentiable manifold and their applications*. Chandrama Prakashan, Allahabad.
3.Sinha, B. B.(1982).*An introduction to modern differential geometry*. Kalyani Publishers, New Delhi.

MAT 403B **FLUID MECHANICS** **Credit-5/ Hours-75**

Course objective: Students will learn concepts related to motion of bodies in fluids and how to model and solve problems related to motion of such bodies.

Learning Outcomes: After studying this course, students will be able to-

1. Describe concepts related to motion in fluid.
2. Compare flows in different types of fluids.
3. Apply the results in solving problems related to fluids.

UNIT I:

Elementary notions of fluid motion: Body forces and surface forces, nature of stresses, Transformation of stress components, Stress invariants, Principal stresses, Nature of strains, Rates of strain components, Relation between stress and rate of strain components, General displacement of a fluid element, Newton's law of viscosity, Navier-Stokes equation (sketch of proof).

UNIT II:

Equation of motion for inviscid fluid, Energy equation, Vortex motion-Helmholtz's vorticity theorem and vorticity equation, Kelvin's circulation Theorem, Mean Potential over a spherical surface, Kelvin's Minimum kinetic energy Theorem, Acyclic irrotational motion.

UNIT III:

Wave motion in a gas. Speed of Sound. Equation of motion of a gas. Subsonic, Sonic and Supersonic flows of a gas. Isentropic gas flows.

UNIT IV:

Normal and oblique shocks. Plane Poiseuille and Couette flows between two parallel plates. Unsteady flow over a flat plate. Reynold's number.

Reference Books:

1. Landau, L. D. and Lifshitz, E. M.(1987). *Fluid Mechanics*(2nd ed.). Butterworth-Heinemann.
2. Curle, N. and Davies, H.J.(1968). *Modern Fluid Dynamics*(Vol. 1).D. Van Nost. Comp London.
3.Yuan, S. W.(1967). *Foundation of Fluid Mechanics*. Prentice-Hall.
4. Ramsey,A. S.(1960). *A Treatise on Hydrodynamics*(Part). I, G. Bell and Sons Ltd.
5. Chalton, F.(1985): *A text book of fluid dynamics*. CBS Publication, New Delhi.

Elective Papers(Optional Papers) (Any one from MAT 404A, MAT 404B and MAT 404C)

MAT 404A **GENERAL RELATIVITY AND COSMOLOGY** **Credit-5/ Hours-75**

Course objective: Students will learn concepts related to general relativity and cosmology and apply these concepts in solving problems related to these fields.

Learning Outcomes: After studying this course, students will be able to-

1. Explain concepts related to General Theory of Relativity.
2. Compare Newtonian Mechanics and Mechanics in curved space time.
3. Apply the results to solve problems related to general relativity.

UNIT I:
Review of special theory of relativity and the Newtonian Theory of Gravitation. Principle of equivalence and general covariance. Geodesic Principle. Newtonian approximation.

UNIT II:
Schwarzchild external solution and its isotropic form. Planetary orbits and analogues of Kepler's law in general relativity. Advance or perihelion of a planet. Bending of light rays in gravitational field. Gravitational redshift of spectral lines.

UNIT III:
Energy momentum tensor of a perfect fluid. Schwarzchild internal solution. Boundary conditions. Energy momentum tensor of an electromagnetic field. Einstein-Maxwell equations. Reissner-Nordstrom solution.

UNIT IV:
Mach's Principle. Einstein modified field equations with cosmological term. Static Cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe. Hubble's law. Cosmological principle's Wey' is postulate. Derivation of Robertson-Walke metric.

Reference Books:

1. Weatherburn, C. E.(1950). *An Introduction To Reimanian Geometry and the tensor Calculus*. Cambridge University Press.
2. Narlikar, J. V. (1978). *General Relativity and Cosmology*. The Macmillan Company of India Ltd.
3. Prakash, S. (2014). *Relativistic Mechanics (16th ed.)*. Pragati Prakashan.
4. Roy, S.R. and Bali, R. (2008). *Theory of Relativity*. Jaipur Publishing house.

MAT 404B **THEORY OF OPTIMIZATION** **Credit-5/ Hours-75**

Course objective: Students will learn concepts related to optimization of convex functions, functions on convex sets, nonlinear programming and their usage in solving problems related to optimization,

Learning Outcomes: After studying this course, students will be able to-

1. Explain various optimizations methods.
2. Compare Linear and nonlinear optimization problems.
3. Apply the optimization methods to solve problems arising in industrial and management fields.

UNIT I:

Linear inequalities and theorems of the alternative, Farka's lemma, Structure of Convex Sets: Algebraic Interior and Algebraic Closure of Convex Sets, Minkowski Gauge Function, Relative Interiors of Convex Sets. Convex polyhedral, Cones, Convex cones.

UNIT II:

Convex function, Continuity of Convex Functions, Epigraph, Conjugates of convex functions, Differentiable convex functions,. Concave function, Convex programming problems and its optimal solutions. Global minimum and local minimum of convex programming problem with constraints.

UNIT III:

Nonlinear Programming: The Fritz John necessary optimality conditions, Constraint Qualifications, The Karush Kuhn-Tucker (KKT) sufficient optimality conditions using convexity of constraint nonlinear programming problems. Applications of Nonlinear Programming.

UNIT IV:

Duality Theory in Nonlinear Programming, Examples of Dual Problems, Duality theorems, Generalized Convexity in Nonlinear Programming. Introduction to Semi-infinite Programming, Applications of Semi-infinite Programming.

Reference Books:

1. Guler,O.(2010). *Foundations of Optimization*. Springer Science+Business Media.
2. Chong, K. P. P. and Zak, S. H.(2003).*An Introduction to Optimization*(3rd ed.). Johan Welly & Sons Inc.
3. Avriel, M., Diewert, W.E., Schaible, S. and Zang, I.(2010). *Generalized Concavity*.SIAM Classics in Applied Mathematics.
4. Cambini, A. and Martein, L.(2009). *Generalized Convexity and Optimization*, Lecture Notes in Economics and Mathematical Systems Vol. 616, Springer-Verlag Berlin Heidelberg.

MAT 404C **MATHEMATICAL MODELING** **Credit-5/ Hours-75**

Course objective: Students will learn concepts related to modeling of real world problems in mathematical form and their solution.

Course Outcomes: After studying this course, students will be able to-

1. Describe Concepts related to Mathematical modeling of real world problems.
2. Compare different types of mathematical models.
3. Apply the methods to solve industrial an management problems by modeling them in mathematical form.

UNIT1:

Simple situations requiring mathematical modeling, techniques of mathematical modeling, Classifications, Characteristics and limitations of mathematical models, Some illustrations.

UNIT II:

Mathematical modeling through differential equations, linear growth and decay models, Non linear growth and decay models, Compartment models, Mathematical modeling in dynamics through ordinary differential equations of first order.

UNIT III:

Mathematical models through difference equations, some simple models, Basic theory of linear difference equations in economics and finance, mathematical modeling through difference eequations in polulation dynamics and genetics

UNIT IV:

Situations that can be modeled through graphs, Mathematical models in terms of Directed graphs, Mathematical models in terms of signed graphs, Mathematical models in terms of weighted graphs. Mathematical modeling through linear programming, linear programming models in forest management. Transportation and assignment models.

Reference Books:

1. Kapoor, J. N. (1988). *Mathematical Modeling*. Wiley Eastern.
2. Burghes, D. N. (1988). *Mathematical Modeling in the Social Management and Life Science*. Ellie Herwood and John Wiley.
3. Charlton, F.(1989). *Ordinary Differential and Difference Equations*. Van Nostrand.

MAT 405 **PROJECT** **Credit-4**

Each student will have to complete a project in fourth semester. It will be of 4 credits. The evaluation of Semester- III and Semester-IV projects will be done together at the end of fourth semester and will consist of 100 marks.

JANANAYAK CHANDRASHEKHAR UNIVERSITY, BALLIA

Department of Mathematics

Minor elective Paper for first semester students of other faculties (faculties other than Science).

Course code (Paper code) - **MAT ME 01**

Course Name - Elementary Number Theory and Calculus Credit-4/ Hours-60/ Marks-100

Course Outcomes: After studying this course, students will be able to-

1. Explain various concepts related to calculus and number theory.
2. Appreciate the method of rigorous mathematical deduction process to obtain results.
3. Apply results in solving problems arising in their field of study.

UNIT I:

Well Ordering Principle, Archimedean Property, First Principle of finite Induction, Binomial Theorem, Division Algorithm, Greatest Common Divisor, Euclid's Lemma, Euclidean Algorithm, Least Common Multiple, The Diophantine Equation of the form $ax + by = c$.

UNIT II:

Prime and Composite numbers, Fundamental Theorem of Arithmetic, Equivalence relations and partitions, Congruence modulo – n and their properties, Binary and Decimal representation of integers, Linear congruence and Chinese Remainder Theorem.

UNIT III:

Functions and Graphs, Elementary functions, Limit and Continuity of functions, Differentiability of functions.

UNIT IV:

Order and degree of differential equations, first order differential equations and their solution, application of first order differential equations.

Reference Books:

1.Burton, D. M. (2011). *Elementary Number Theory* (7th ed.). McGraw Hill.
2. Lal, R. (2002). *Algebra* (Vol. I). Shail Publications, Allahabad.
3. Malik, S. C. and Arora, S. (2017). *Mathematical Analysis* (5th ed.). New Age Int. Pub.
4. Rai, B. and Chaudhary, D. P. (2005). *Ordinary Differential equations*. Narosa Pub.